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Interactive Multiobjective Optimization Procedure

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This research focuses on multiobjective system design and optimization. The primary goal is to develop and test a mathematically rigorous and efficient interactive multiobjective optimization algorithm that takes into account the designer's preferences during the design process. In this research, an interactive multiobjective optimization procedure (IMOOP) that uses an aspiration-level approach to generate Pareto points is developed. This method provides the designer or the decision maker (DM) with a formal means for efficient design exploration around a given Pareto point. More specifically, the procedure provides the DM with the Pareto sensitivity information and the Pareto surface approximation at a given Pareto design for decision making and tradeoff analysis. The IMOOP has been successfully applied to two test problems. The first problem consists of a set of simple analytical expressions for its objective and constraints. The second problem is the design and sizing of a high-performance and low-cost 10-bar structure that has multiple objectives. The results indicate that the Pareto designs predicted by the Pareto surface approximation are reasonable and the performance of the second-order approximation is superior compared to that of the first-order approximation. Using this procedure a set of new aspirations that reflect the DM's preferences are easily and efficiently generated, and the new Pareto design corresponding to these aspirations is close to the aspirations themselves. This is important in that it builds the confidence of the DM in this interactive procedure for obtaining a satisfactory final Pareto design in a minimal number of iterations.

Introduction

LARGE-SCALE engineering design problems, such as an aircraft design or an automobile design, are multidisciplinary in nature as they often involve smaller, more tractable disciplines, which are intrinsically linked to one another by interdisciplinary interactions. An important issue that complicates the design and optimization of such large systems is that each participating discipline or design team is frequently required to maximize/minimize its own design goals while satisfying design constraints. As a result large-scale design problems are multiobjective in nature.

For illustration, consider a typical automobile design problem. Some of the participating disciplines are aerodynamics, elastic structures, occupant dynamics, fuel economy, etc. Many of these disciplines have their own design objectives to meet. The aerodynamics has an objective of minimizing the drag, the elastic structures has an objective of minimizing the mass, whereas the fuel economy has an objective of maximizing the fuel efficiency, etc. These design goals are potentially conflicting requirements reflecting the technical and economical performances of a given system design. To accommodate these conflicting design goals (or objectives) and to explore the design options, one needs to formulate the optimization problem with multiple objectives (vector optimization). The vector optimization algorithms and procedures have to arrive at an optimum design that attains these objectives as closely as possible while satisfying constraints strictly. This calls for the incorporation of Pareto optimality concepts into optimization algorithms and procedures that require the designer's involvement as a decision maker (DM).[1–3] In the next section, the multiobjective optimization problem statement is given.

Problem Formulation

A general multiobjective optimization problem is to find the design variables that optimize a vector objective function $[\boldsymbol{F}(\boldsymbol{x}) = \{f_1, \ldots, f_m\}]$ over the feasible design space. It is the determination of a set of nondominated solutions (Pareto optimum solutions or noninferior solutions) that achieves a compromise among several different objective functions. The problem formulation in standard form is as follows:

$$
\begin{aligned}
&\text{minimize} \quad F(\boldsymbol{x}) = \{f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_m(\boldsymbol{x})\} \\
&\text{subject to:} \quad h_j(\boldsymbol{x}) = 0, \qquad j = 1, \ldots, q \\
&\qquad g_k(\boldsymbol{x}) \geq 0, \qquad k = 1, \ldots, p \\
&\qquad x_l^u \geq x_l \geq x_l^l, \qquad l = 1, \ldots, n
\end{aligned}
\tag{1}
$$

where f_i, h_j, and g_k map R^n into R and the region $S = \{\boldsymbol{x} \mid \boldsymbol{x}^l \leq \boldsymbol{x} \leq \boldsymbol{x}^u\}$ defined by Eq. (1) is assumed to have a nonempty interior and may be unbounded. It will also be assumed that the functions f_i, h_j, and, g_k are twice differentiable.

Pareto Optimal Concept

The Pareto optimality concept is stated as follows: A vector of $\boldsymbol{x}^*$ is Pareto optimal if there exists no feasible vector $\boldsymbol{x}$ that would decrease some objective function without causing a simultaneous increase in at least one objective function. Mathematically, the Pareto optimality is defined as follows: A vector of $\boldsymbol{x}^*$ is a Pareto optimum iff, for any $\boldsymbol{x}$ and i,

$$
f_j(\boldsymbol{x}) \leq f_j(\boldsymbol{x}^*), \qquad j = 1, \ldots, m; \; j \neq i, \qquad \Longrightarrow f_i(\boldsymbol{x}) \geq f_i(\boldsymbol{x}^*)
\tag{2}
$$

This concept can be explained graphically. Consider a two-design-variable system that has two design objectives, i.e., $m = 2$, and two constraints, which are shown in Fig. 1. The shaded region to the left of the constraint surface is feasible. The corresponding feasible space and the Pareto set in objective function space is shown in Fig. 2. The solid lines labeled "Pareto Set" represent all of the noninferior solutions that are possible for this design problem. Thus, choosing one design over the other on the Pareto set calls for the decision-making problem by considering the preferences of the DM. Note that the Pareto set includes the individual optima of each objective function at its corners. The Pareto set, in the present case and in general, is nonlinear and contains both C^0 and C^1 discontinuities. This is due to the nature of the design space and the active constraint switching. In the next section an optimization method for obtaining these Pareto points is described.

Pareto Point Generation Scheme

The Pareto set can be defined with the help of a so-called ideal point (or utopia point) f^u (Fig. 2). This is a vector that contains the individual optima (or an estimate of individual optima) of each of

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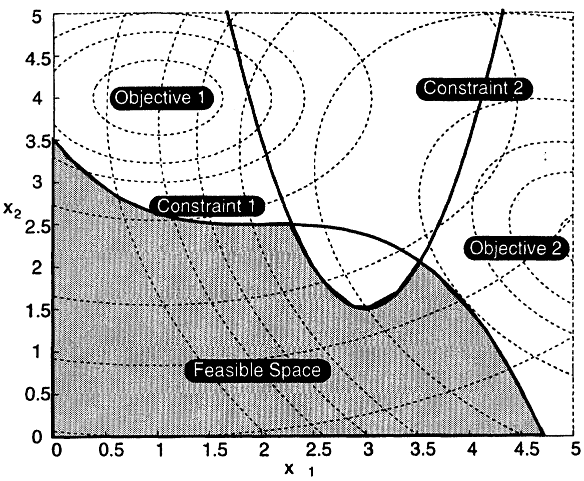

Fig. 1 Design variable space.

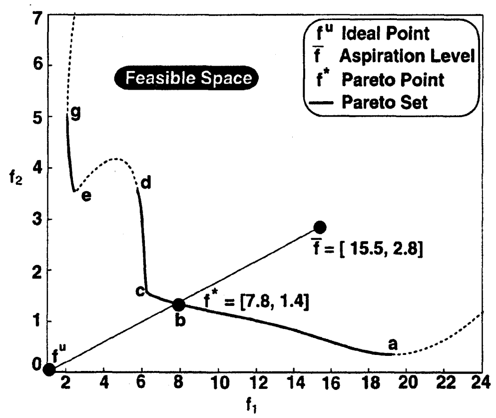

Fig. 2 Objective function space.

the objective functions. Note that this f^u, which is not achievable, is given such that the following condition is satisfied:

$$f_i(x^*) > f_i^u, \qquad i = 1, \ldots, m$$

There are a number of procedures that are proposed in literature for the generation of a compromise point. The approach that is being followed here is based on minimization of the distance between a feasible point and the ideal point and is called compromise programming (CP).[4] There are a number of metrics, i.e., L_2 norm, $L_{\inf}$ norm, etc., that can be used to measure this distance. In this implementation, it is measured in terms of the weighted Tchebyshev norm.[5] The optimization problem in standard form for the weighted Tchebyshev norm is

$$\begin{aligned} &\text{minimize} \quad q \\ &\text{subject to:} \quad g_j \geq 0, \qquad j = 1, \ldots, p \\ &\qquad h_k = 0, \qquad k = 1, \ldots, q \\ &\qquad x^u \geq x \geq x^L \end{aligned} \tag{3}$$

where

$$q = \max w_i\big(f_i(x) - f_i^u\big), \qquad i = 1, \ldots, m$$

The advantage of the Tchebyshev norm approach is that efficient solutions are produced even when the Pareto set is nonconvex. However, the given formulation of Tchebyshev norm has two disadvantages: 1) The solutions obtained may be only weak Pareto optimal, i.e., there may be other solutions that improve a criterion, whereas the others are fixed; and 2) the objective function q, also called an achievement function, is nonsmooth. To avoid these two undesirable features, the following equivalent problem with augmented Tchebyshev norm[5] is solved:

$$\begin{aligned} &\text{minimize} \quad z + \alpha \sum_{i=1}^{m} w_i f_i(x) \\ &\text{subject to:} \quad z \geq w_i\big(f_i(x) - f_i^u\big), \qquad i = 1, \ldots, m \\ &\qquad g_i \geq 0, \qquad i = 1, \ldots, p \\ &\qquad h_j = 0, \qquad j = 1, \ldots, q \\ &\qquad x^u \geq x \geq x^L \end{aligned} \tag{4}$$

where α is set as a sufficiently small positive number, for example, 10^{-6}. In the current implementation the parameter w_i is replaced using the ratio

$$w_i = \frac{1}{\bar{f}_i - f_i^u}$$

where $\bar{f}$ has the property

$$\bar{f}_i > f_i^u, \qquad i = 1, \ldots, m$$

The components of $\bar{f}$ are called the aspiration levels (Fig. 2). These are set by the DM prior to solving the CP problem. These aspirations represent the outcomes that the DM considers potentially satisfactory. These aspiration levels can only be achieved when they are on the compromise (or the Pareto) set. In other cases, a Pareto point that best represents the aspiration levels is obtained. Note that different Pareto points can be generated for different settings of the aspiration levels.

In this research, the CP method is used for the generation of an initial Pareto design. If the DM is not satisfied with the current Pareto design, then there is interest in exploring the Pareto surface that is in the vicinity of the current Pareto point. In this research, a Pareto sensitivity analysis and a second-order Pareto surface approximation around the current Pareto design are proposed to assist and aid the DM in exploring the Pareto surface, in conducting the tradeoff (or sensitivity) studies, and in specifying new aspiration levels that are achievable (those that are on the Pareto surface).

Multiobjective Optimization Methods

A variety of techniques and applications of multiobjective optimization have been developed over the past few years. The progress in the field of multicriteria optimization was summarized by Hwang and Masud[6] and later by Stadler and Dauer.[2] Note that, in this presentation, the terms *objective function* and *criterion* are used interchangeably. Rao et al.[7] treated several different structural and structural control problems by applying a weighting method, a goal programming method, and a cooperative game theory in which the objectives are fixed a priori. Dovi and Wrenn[8] have compared the performances of envelope function formulation (KSOPT), global criterion formulation, and utility function formulation in the context of a typical wide-body subsonic transport aircraft configuration. An approach called the compromise decision support problem (DSP), which is derived from goal programming, has been developed by Mistree et al.[9] and is used for ship design. In the compromise DSP, the lexicographic minimization concept has been implemented. Hajela and Shih[10] proposed a minimum variant of the global criterion approach to obtain solutions to multiobjective optimum design problems involving a mix of continuous, discrete, and integer design variables. In Ref. 11, a fuzzy logic scheme, where objectives are ranked as being either soft or hard, is used. Messac[12] proposed a method called physical programming, which captures the DM's preferences a priori in a consistent manner to avoid specifying the objective weightings. For each objective function, this method requires the DM to specify bounds for various satisfactory levels. Based on the bounds specified, utility functions that reflect the DM's preferences are constructed. Tappeta and Renaud[13] proposed different formulations of multiobjective collaborative optimization for multiobjective nonhierarchic system design and optimization. These formulations also require that the weights (or the DM's preferences) are known a priori.

Most of the methods mentioned involve converting a multiobjective problem into a single-objective problem and then solving this single objective optimization for a compromise solution. This scalarization is usually achieved by using either the weights or the targets that the DM has to specify a priori. Some of the disadvantages of these methods are the following:

1) The methods require a priori selection of weights or targets for each of the objective functions.

2) The methods provide information for only one design scenario, i.e., a single Pareto solution.

3) The methods are unable to generate proper Pareto points for nonconvex problems, e.g., the weights method.

4) The methods are incapable of generating sensitivity information for tradeoff and decision making.

5) There are no inherent capabilities for design exploration.

There is another class of methods that are population based and use evolutionary or genetic algorithms (GAs) for the generation of the Pareto set. An overview of these evolutionary methods is presented by Fonseca and Fleming.[14] Because the GA searches using a population of points, rather than using a point-to-point search, it is possible to generate a large number of Pareto points. The multiobjective GAs also require a scalar function (or a fitness function) to be defined such that the Pareto designs have a higher fitness value compared to the non-Pareto designs. Some of the applications of the multiobjective GAs include those by Goldberg,[15] Obayshi,[16] Yoo and Hajela,[17] and Crossley et al.[18] The multiobjective GAs are computationally expensive because they attempt to generate a large number of Pareto points.

Realizing the limitations of the traditional methods, there are a number of interactive methods that have been proposed in the operational research community for multicriterion decision making that attempt to address some of the issues mentioned earlier. These interactive methods rely on the preference information generated interactively during the optimization from the DM. These procedures are characterized by phases of decision making alternating with phases of optimization. Developments and applications of interactive multiobjective optimization procedures in engineering have been much fewer. Three of the interactive methods that have been applied for multiobjective engineering system design are mentioned as follows.

Nakayama and Furukawa[19] have introduced the so-called satisficing tradeoff method, which is based on CP. In this approach, if the DM is not satisfied with the current Pareto point, new aspirations must be set for only those objectives that have to be improved and the program sets the other aspirations based on the linearization. Yang and Sen[20] proposed an interactive method, which consists of an interactive step tradeoff method[21] and the estimation of local utility functions. Here, the DM specifies the amount of sacrifice to be made in some objectives and also specifies the objectives that have to be improved. Based on this information an auxiliary optimization problem, i.e., the CP problem, is solved to achieve a new Pareto point that meets the DM's requirements. After generating a sufficient number of Pareto points, linear local utility functions that capture the DM's preferences are used to arrive at a final Pareto design. Diaz[22] has proposed an interactive method that is based on the postoptimal parameter sensitivity information. This method provides the DM with sensitivity information of objective functions with respect to current levels of aspirations for decision making.

These interactive methods empower the designer to act as a DM at the end of each optimization. They provide the designer with more information at each Pareto design than what the traditional methods mentioned earlier provide. However, these methods are limited because they do not provide any information about the available Pareto designs in the neighborhood of the current Pareto design. More specifically, these methods do not provide the Pareto sensitivity information and do not approximate the Pareto surface around the current design.

Keeping these limitations of the mentioned interactive methods in mind, the goal of this research is to develop an interactive method that provides the DM with the following information during each phase of decision making: 1) Pareto sensitivity information, 2) Pareto surface representation using response surfaces, 3) tradeoff analysis and decision making, and 4) quantification of the qualitative decisions made by the DM.

These four capabilities provide the DM with a formal means for design exploration around the current Pareto point. The original contributions of this research to the multiobjective optimization community are the Pareto sensitivity analysis, an efficient method for generating the approximate Pareto points around the current Pareto design, and the approximate Pareto surface representation using first- and second-order response surfaces. Each one of these concepts is discussed in detail in the following sections. An interactive multiobjective optimization procedure (IMOOP), which utilizes the preceding concepts, is also developed. In the Results section, the IMOOP is implemented in application to two design problems.

Pareto Sensitivity Analysis

As mentioned earlier, the DM explores the Pareto set by varying the aspiration levels. The Pareto sensitivity analysis can improve the efficiency of any scheme that requires this type of user interaction. The objective here is to provide the sensitivity information that can potentially reduce the number of interactions needed before a satisfactory solution is found. This information can be used for 1) updating the aspiration levels (in CP), 2) generating the first-order Pareto surface approximation, 3) conducting the tradeoff analysis and decision making, and 4) checking the robustness of the current design.

The sensitivity analysis at a given Pareto point provides the variation in one objective given the variation in another objective, i.e., $\mathrm{d}f_i/\mathrm{d}f_j$, on the Pareto surface in a given direction. These sensitivities, which are tangent lines to the Pareto surface, are directional derivatives. Note that the Pareto surface, at a given Pareto point, has infinite tangent directions. The sensitivity analysis presented seeks to find the sensitivity information along the principal directions, i.e., feasible descent direction of each of the objectives, in the objective function space.

Apart from the approach by Hernandez,[23] the authors are not aware of any other method in the literature for conducting this sensitivity analysis. The approach by Hernandez[23] is illustrated for a two-dimensional problem and its extension/implications to multiobjective optimization are not discussed. In this research, a new and simple approach based on the active constraint set strategy is developed. It is assumed here that the active constraint set remains active for a small move, in any direction, along the Pareto surface from the current point. As a result, the sensitivities predicted in a given direction using this method are valid only if the active constraint set remains unchanged.

At a given constrained Pareto optimal point $\boldsymbol{x}^*$, let $g^a(\boldsymbol{x}^*)$ represent the set of active constraints (including the active bounds) and $\boldsymbol{J}$ represent the Jacobian of the active constraint set at $\boldsymbol{x}^*$. The terms of the Jacobian are given by

$$\boldsymbol{J} = \nabla_x g^a \tag{5}$$

The projection matrix $\boldsymbol{P}$ of the active constraint set is

$$\boldsymbol{P} = \boldsymbol{I} - \boldsymbol{J}^T(\boldsymbol{J}\boldsymbol{J}^T)^{-1}\boldsymbol{J} \tag{6}$$

where $\boldsymbol{P}$ is the projected feasible directions along which the constraint set status is unchanged. Note that this projection matrix is the tangent hyperplane to the active set at $\boldsymbol{x}^*$. A feasible direction with the greatest improvement of objective f_i is obtained by projecting $\nabla_x f_i$ onto matrix $\boldsymbol{P}$:

$$\boldsymbol{d}_{f_i}(n\boldsymbol{x}1) = \boldsymbol{P}(-\nabla_x f_i), \qquad i = 1, \ldots, m \tag{7}$$

To determine the Pareto sensitivity $\mathrm{d}f_i/\mathrm{d}f_l$ along the feasible descent direction of objective f_l, differentiate f_i and f_l with respect to $\boldsymbol{x}$:

$$\mathrm{d}f_l = \boldsymbol{d}_{f_l}^T\,\mathrm{d}\boldsymbol{x} \tag{8}$$

$$\mathrm{d}f_i = \boldsymbol{d}_{f_i}^T\,\mathrm{d}\boldsymbol{x}, \qquad i = 1, \ldots, m \tag{9}$$

solving Eq. (8) for $\mathrm{d}\boldsymbol{x}$ by projection,

$$\mathrm{d}\boldsymbol{x} = \left[\boldsymbol{d}_{f_l}^T\boldsymbol{d}_{f_l}\right]^{-1}\boldsymbol{d}_{f_l}^T\,\mathrm{d}f_l \tag{10}$$

Equation (10) represents the change in design variables along the feasible descent direction of objective f_l for a decrease in the objective, i.e., $\mathrm{d}f_l$. By substituting the result in Eq. (9) for $\mathrm{d}\boldsymbol{x}$ and by rearranging the terms, the following expression for the tradeoff information of objective f_i along the feasible descent direction of objective f_l, $(\mathrm{d}f_i/\mathrm{d}f_l)$, is obtained:

$$\frac{\mathrm{d}f_i}{\mathrm{d}f_l} = \boldsymbol{d}_{fi}^T\left[\boldsymbol{d}_{fl}^T\boldsymbol{d}_{fl}\right]^{-1}\boldsymbol{d}_{fl}, \qquad i = 1, \ldots, m \tag{11}$$

Similarly, one can conduct a Pareto sensitivity analysis along the feasible descent direction of each of the other objective functions. This sensitivity information can be represented in a matrix form, called the tradeoff matrix, whose ith row represents the tradeoffs required for an improvement in the ith objective (f_i):

$$T = \begin{bmatrix} 1 & \dfrac{\mathrm{d}f_2}{\mathrm{d}f_1} & \cdots & \dfrac{\mathrm{d}f_m}{\mathrm{d}f_1} \\ \dfrac{\mathrm{d}f_1}{\mathrm{d}f_2} & 1 & \cdots & \dfrac{\mathrm{d}f_m}{\mathrm{d}f_2} \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{\mathrm{d}f_1}{\mathrm{d}f_m} & \dfrac{\mathrm{d}f_2}{\mathrm{d}f_m} & \cdots & 1 \end{bmatrix} \tag{12}$$

As mentioned at the beginning of this section, there are a number of different tangent directions along which the Pareto sensitivity information can be represented. The sensitivity information given in Eq. (11) is along the feasible descent direction of each of the objective functions. Another way of representing the Pareto sensitivity information is to find $\mathrm{d}f_i/\mathrm{d}f_l$ while keeping the other objectives fixed at their current values. The procedure for determining this information is the same as the one derived earlier except that the Jacobian matrix [Eq. (5)] is modified to include not only the gradients of each of the active constraints but also the gradients of each of the objective functions that are kept constant as

$$\boldsymbol{J} = \begin{bmatrix} \nabla_x g_j^a \\ \nabla_x f_k \end{bmatrix}, \qquad j = 1, \ldots, p+q, \quad k = 1, \ldots, m, \quad k \neq i, l \tag{13}$$

The tradeoff between any two objective functions exists only if the corresponding off-diagonal element in the tradeoff matrix is negative. The magnitude of the tradeoff, if it exists, is given by the absolute value of the corresponding off-diagonal element. On the other hand, a positive element in any given row indicates that there is no tradeoff between these two objectives, i.e., both of them can be improved simultaneously. Note that at least one element in any given row has to be negative for a given design to be a local Pareto optimum. This condition is necessary and not sufficient for a local Pareto optimum. The Pareto sensitivity information can be used to find the normal to the Pareto surface, which defines the tangent hyperplane to the Pareto surface at the current Pareto design.

Pareto Surface Representation

As mentioned earlier, the sensitivity analysis just presented provides information in a few specified directions. The goal of this section is to represent the Pareto surface in the neighborhood of the current Pareto design by using the first- and second-order approximation techniques. This representation is useful in decision making because it provides the comprehensive information about the Pareto surface and facilitates the DM to explore other Pareto designs. The first-order terms of the Pareto surface approximation are determined based on the tradeoff matrix T given in Eq. (12), and the second-order (or the Hessian) terms are determined based on the Pareto data generated around the current design.

Before describing these approximations, it is necessary to mention the limitations and conditions under which they are valid. As mentioned earlier, the Pareto surface is highly nonlinear, is nonsmooth, and has discontinuities. For smooth approximations to work, the active set changes should be mild (C^0 discontinuities), and for them to be valid in the entire neighborhood of the current point, there should be no nearby jumps or kink (C^1 discontinuities) in the Pareto surface. The mathematical description of the Pareto surface in objective function space is given by

$$S(f_1, f_2, \ldots, f_m) = 0 \tag{14}$$

The first-order approximation to the Pareto surface at the current Pareto design ($\boldsymbol{x}^*$) is given by

$$\tilde{S} = \sum_{i=1}^{m} \frac{\mathrm{d}S}{\mathrm{d}f_i}(f_i - f_i^*) = 0 \tag{15}$$

$$\tilde{f}_m = f_m^* + \sum_{i=1}^{m-1} \frac{\mathrm{d}f_m}{\mathrm{d}f_i}(f_i - f_i^*) \tag{16}$$

The gradient terms in Eq. (16) are the normals to the Pareto surface at the current Pareto point. The $\mathrm{d}f_m/\mathrm{d}f_i$ is the change in objective f_m for a given change in f_i while the other objectives are kept constant, i.e., the second type of Pareto sensitivity information mentioned in the preceding section. The second-order approximation to the Pareto surface is given by

$$\tilde{f}_m = f_m^* + \sum_{i=1}^{m-1} \frac{\mathrm{d}f_m}{\mathrm{d}f_i}\Delta f_i + \frac{1}{2}\sum_{i,j=1}^{m-1} \Delta f_i \boldsymbol{H}_{ij} \Delta f_j \tag{17}$$

where $\boldsymbol{H}_{ij}$ is given by

$$\boldsymbol{H}_{ij} = \frac{\mathrm{d}^2 f_m}{\mathrm{d}f_i \mathrm{d}f_j} \tag{18}$$

Note that there are m objective functions, and the second-order approximation [Eq. (17)] represents one of the objectives (mth objective) in terms of the other $m-1$ objectives. If one were to neglect off-diagonal elements and estimate only the diagonal terms of the Hessian matrix, the number of terms to be determined would be $m-1$. On the other hand, if one were to estimate the entire symmetric Hessian matrix, the number of terms to be determined would be $m(m-1)/2$. Note that the Pareto data to be generated (or required) are proportional to the number of terms to be determined. The decision on whether to neglect off-diagonal terms or not should be made based on the complexity of the Pareto set and the cost of obtaining the extra Pareto data. The second-order terms are determined based on Pareto data generated around the current design, which is explained next.

Pareto Data Generation

One obvious way of generating the required Pareto data is to solve a number of CP problems with different aspiration levels around the current Pareto design. This involves the solving of a full optimization problem [CP; Eq. (4)] a number of times. This is prohibitively expensive because it requires a large number of analysis calls for function and gradient evaluations. To avoid solving a large number of expensive CP problems, in this research, approximate Pareto points are generated around the current Pareto design by using a projection method that uses the gradient information available at the current Pareto design. The details of this method are given later.

The projection method essentially involves the finding of approximate Pareto points $\boldsymbol{f}^{i*}$ by projecting the aspiration levels $\bar{\boldsymbol{f}}^i$ onto the Pareto surface, as shown in Fig. 3. These aspiration levels are chosen such that they are on the first-order Pareto surface [Eq. (16)] within a certain percentage from the current Pareto design. This is done in two steps for each set of the aspiration levels $\bar{\boldsymbol{f}}^i$. In the first step, the following approximate CP problem is solved to obtain a Pareto point corresponding to aspirations (for example, $\bar{\boldsymbol{f}}^1$):

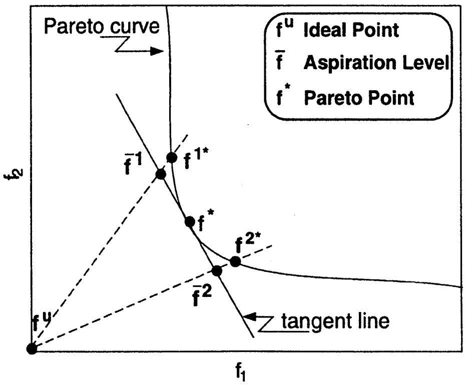

Fig. 3 Projecting points $\bar{f}^1, \bar{f}^2$ onto the Pareto curve.

$$\text{minimize} \quad z + \alpha \sum_{i=1}^{m} w_i \frac{\tilde{f}_i(x)}{\bar{f}_i^1 - f_i^u}$$

$$\text{subject to:} \quad z \geq \frac{[\tilde{f}_i(x) - f_i^u]}{\bar{f}_i^1 - f_i^u}, \qquad i = 1, \ldots, m$$

$$\tilde{g}_j(x) \geq 0, \qquad j = 1, \ldots, p \tag{19}$$

$$\tilde{h}_k(x) = 0, \qquad k = 1, \ldots, q$$

$$\Delta x_l^u \geq \Delta x_l^l \geq \Delta x_l, \qquad l = 1, \ldots, n$$

where $\tilde{f}_i(x)$, $\tilde{g}_j(x)$, and $\tilde{h}_k(x)$ are the approximations of $f_i(x)$, $g_j(x)$, and $h_k(x)$ with respect to the design variables at the current Pareto design x^*. In this research, only the first-order approximations of $f_i(x)$, $g_j(x)$, and $h_k(x)$ with respect to the design variables are assumed to be available at the current design. As a result, the preceding approximate CP problem becomes a linear programming (LP) minimization. Note that the optimum objective function values, at the end of this step, will only be equal to the aspiration levels if there are no active constraint set switches. The optimum design (in x space) obtained at the end of this step is used as the starting design during the second step, which is explained next.

In the second step, the approximate Pareto point f^{1*} corresponding to aspirations $\bar{f}^1$ is found by projecting the optimum design obtained in step 1 onto the Pareto surface. This is achieved by solving a few iterations of the compromising programming problem [Eq. (4)] with aspirations $\bar{f}^1$ using the generalized reduced gradient (GRG) method.[24] As mentioned earlier, the computational effort is minimized in this research by using the gradient information (with respect to x) available at the current Pareto design x^* during the projection. Note that, because the gradient information from the current Pareto design is used, the analysis routines are invoked by the optimizer only for function evaluation.

Tradeoff Analysis and Decision Making

If a given design is Pareto optimal, there is no other feasible solution that improves all of the objective functions. Therefore, if the DM wants to improve some of the objective functions it can only be done at the expense of other objective functions. The tradeoff analysis considers or addresses the issue of how much one has to relax some objectives for compensating for the improvement of other objectives. In large design problems it is often difficult for the DM to know precisely how much to change a given aspiration level to obtain a satisfactory solution. The Pareto sensitivity information and the Pareto surface approximation presented earlier are used to guide the DM in setting the amount of relaxation or improvement of a given objective function (decision making or aspiration level setting). The goal of the next section is to address this issue.

Generating the New Aspiration Levels

If the DM is not satisfied with the current Pareto point, based on the current values of objective functions and tradeoff values [Eq. (12)] it is possible for the DM to make some qualitative decisions regarding which objective functions to be further improved, which objective functions can be relaxed, and which objective functions are acceptable as they are. To generate a Pareto point that meets these preferences, the DM is required to specify aspirations, i.e., quantitative decisions. In real-life design problems the number of objectives can be large, and the DM might not be able to set new aspiration levels for each objective function. In many cases the DM may want to set aspirations for some of the objective functions and have a procedure that generates, automatically, aspirations for the other objectives such that preferences are satisfied.

If the DM sets these aspirations without any knowledge of the Pareto surface (heuristic setting), they may be inconsistent and even unattainable, resulting in a Pareto point that is not satisfactory. It should be noted that the only aspirations that are attainable are the ones that lie on the Pareto surface. As a result, the aspirations that satisfy the DM's preferences and lie on the Pareto surface are the designs that are both satisfactory and attainable. These can be achieved efficiently by solving a suitable optimization problem (that reflects

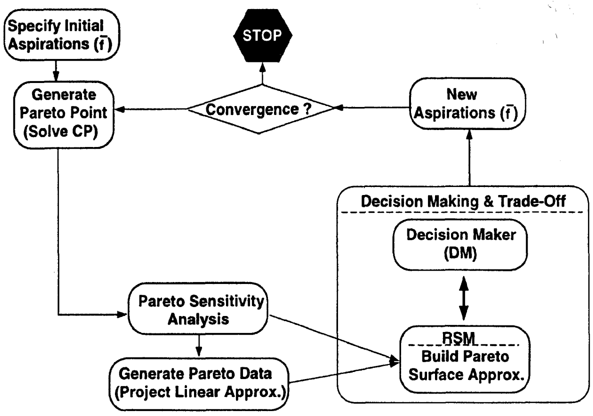

Fig. 4 Flowchart of IMOOP.

the DM's preferences) subject to approximate Pareto surface constraint in objective function space. To summarize, the Pareto sensitivity analysis and the second-order Pareto surface approximation are used for the following purposes:

1) Provide Pareto surface exploration capabilities to the DM.
2) Check for the consistency in aspirations set by the DM.
3) Automatically generate aspirations for some objectives based on the ones set for the other objectives.

IMOOP

This section presents a procedure for the multiobjective optimization in which the DM is directly involved in decision making and is responsible for terminating the procedure. The key elements in this procedure are 1) sensitivity analysis, 2) Pareto point generation scheme, 3) Pareto surface representation, and 4) tradeoff analysis and decision making.

The preceding concepts have been described in detail in the earlier sections of this paper. The flowchart for this procedure is given in Fig. 4. The procedure starts with an ideal point and initial aspiration levels specified by the DM. The Pareto optimal design corresponding to these aspirations is generated by solving a CP problem [Eq. (4)]. At the current Pareto design, the Pareto sensitivity analysis [Eq. (11)] is conducted, and the approximate Pareto design points in the neighborhood of the current design are generated using the projection formulation. These design points coupled with the Pareto sensitivity information are used to form the second-order Pareto surface approximation at the current design [Eq. (17)]. Now the DM is presented with the current Pareto point, the sensitivity information, and the approximate Pareto surface to conduct tradeoff and decision making. If the DM is satisfied with the current Pareto design the procedure is terminated. On the other hand, if the DM is not satisfied and wants to change preferences, this can be done by querying the Pareto surface approximation. Once the DM specifies preferences, a new set of aspirations that reflect the preferences are obtained such that they are on the approximate Pareto surface [Eq. (17)]. Finally, the Pareto point corresponding to these aspirations is generated and presented to the DM. The entire procedure is repeated until the DM is satisfied. Note that the procedure does not have any mathematical basis for its convergence, which is typical of any IMOOP. All of the Pareto designs obtained are potential candidates for the final selection and are, therefore, stored in a database. The DM can always go back to any previously generated design to explore the design space around it.

Results

In this section the IMOOP is implemented in application to two design problems. The first problem consists of a set of simple analytical expressions for its objective and constraint functions. This problem is chosen to illustrate the steps involved in the IMOOP application. The second problem is the design and sizing of a high-performance and low-cost 10-bar structure that has weight, sustainable loads, and displacement as its objective functions. The IMOOP is implemented in MATLAB™ 5.0 (Ref. 25). The sequential quadratic programming[26] is being used for solving the CP

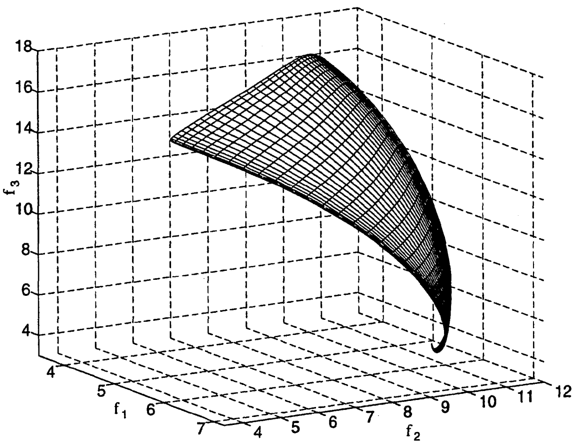

Fig. 5 Pareto surface.

problems, and the GRG method[24] is being used for projecting the aspirations from the tangent hyperplane onto the Pareto surface (Fig. 3).

Test Problem 1: Analytic Example

This problem has three design variables, three objective functions, and a constraint. It is chosen so that the steps and the effectiveness of IMOOP can be demonstrated graphically. The problem definition in standard form is

$$\text{minimize} \quad F(x) = \{f_1(x), f_2(x), f_3(x)\}$$

$$\text{subject to:} \quad g_1(x) = 12 - x_1^2 - x_2^2 - x_3^2 \geq 0 \tag{20}$$

$$x \geq 0$$

where the objective functions are given by

$$f_1 = 25 - \left(x_1^3 + x_1^2(1 + x_2 + x_3) + x_2^3 + x_3^3\right)/10$$

$$f_2 = 35 - \left(x_1^3 + 2x_2^3 + x_2^2(2 + x_1 + x_3) + x_3^3\right)/10$$

$$f_3 = 50 - \left(x_1^3 + x_2^3 + 3x_3^3 + x_3^2(3 + x_1 + x_2)\right)/10$$

From these expressions it can be observed that the objectives are nonlinear functions of design variables, and the constraint is a quadratic function of the design variables. The Pareto surface for this problem is shown in Fig. 5. The following are the steps involved in application of IMOOP to this three-design-variable problem.

Step 1

The first step in the procedure is to choose an ideal point and the initial aspiration levels that the DM wishes to attain. These values have to be chosen based on the experience of the DM. In the present case ideal and initial aspiration points are set to

$$f^u = [0.0 \quad 0.0 \quad 0.0], \qquad \bar{f} = [16.0 \quad 24.0 \quad 40.0]$$

Step 2

The second step is to obtain a Pareto point that best represents the DM's preferences, i.e., the aspirations. This is achieved by solving the CP problem. The Pareto design is given by

$$x^* = [1.939 \quad 2.555 \quad 1.775], \qquad f^* = [5.672 \quad 8.508 \quad 14.179]$$

From the results, it can be observed that the optimum design obtained is better than the aspiration levels specified for each one of the objectives. In many cases, this happens because the DM is not aware of the kind of solutions that can be obtained prior to optimization.

Step 3

Once the optimum design is obtained, it is assumed here that the DM would like to explore the Pareto surface in the neighborhood of the current Pareto design. In this step, the Pareto sensitivity analysis is conducted at the current design to study the effect of changes in one objective function on the other objectives. Note that, at the optimum, the constraint g_1 is active and is assumed to be active for all of the Pareto sensitivity calculations. Based on Eq. (11) the tradeoff matrix is obtained as

$$T = \begin{bmatrix} 1.000 & -1.074 & -1.431 \\ -0.212 & 1.000 & -0.597 \\ -0.194 & -0.411 & 1.000 \end{bmatrix} \tag{21}$$

where the ith row indicates the sensitivity information with respect to objective function f_i along its feasible descent direction. From the tradeoff matrix, it can be seen that a decrease in any objective function from its current value is associated with a simultaneous increase in the other two objective functions, i.e., negative off-diagonal elements. The fact that each of the off-diagonal elements is negative indicates that the current point is Pareto optimal.

Step 4

Upon the completion of sensitivity analysis, the next step is to represent the Pareto surface, around the current design, using the second-order approximation. The first-order approximation to the Pareto surface, which is determined based on the tradeoff matrix T [Eq. (21)], is given by

$$\tilde{f}_3 = 14.179 - [2.683 \quad 1.165] \begin{bmatrix} f_1 - 5.672 \\ f_2 - 8.508 \end{bmatrix} \tag{22}$$

Note that this first-order Pareto approximation is nothing but a tangent plane to the Pareto surface at the current design. In Fig. 6 the first-order approximation is plotted along with the actual Pareto surface. It can be seen from Fig. 6 that the Pareto surface representation is accurate around the current design. This, in general, is the case unless the Pareto surface is nonsmooth, i.e., active set changes around the current design.

To obtain the second-order Pareto surface, it is necessary to generate Pareto data around the current design. Note that, in the present case, at least three Pareto points are needed to obtain the second-order approximation. These points are generated by projecting the aspirations that are on the first-order approximation [Eq. (22)] onto the Pareto surface. The four aspiration points are chosen such that the values of $\bar{f}_1$ and $\bar{f}_2$ are at the corners of a 20% perturbed hypersquare around the current Pareto design. The values of f_3 for these four aspiration points are chosen such that they lie on the first-order Pareto surface. The second-order Pareto surface approximation at the current Pareto design is given by

$$\tilde{f}_3 = 14.179 - [2.683 \quad 1.165] \begin{bmatrix} f_1 - 5.672 \\ f_2 - 8.508 \end{bmatrix}$$

$$- \frac{1}{2} \begin{bmatrix} f_1 - 5.672 \\ f_2 - 8.508 \end{bmatrix}^T \begin{bmatrix} -0.928 & -0.413 \\ -0.413 & -0.412 \end{bmatrix} \begin{bmatrix} f_1 - 5.672 \\ f_2 - 8.508 \end{bmatrix} \tag{23}$$

In Fig. 7, the accuracy of the second-order approximation is compared with the actual Pareto surface. It can be seen from Fig. 7 that the second-order approximation fits the Pareto surface over a wider

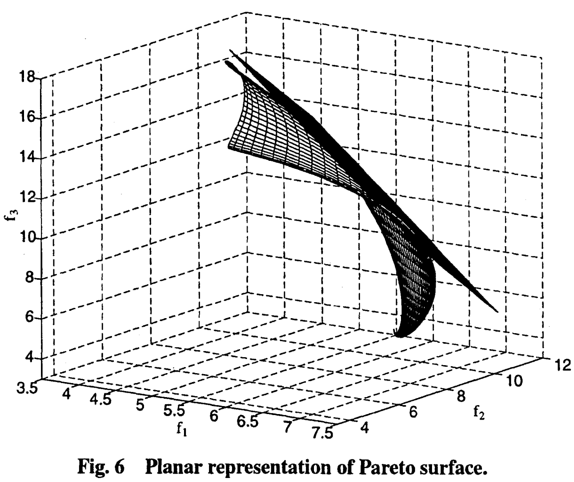

Fig. 6 Planar representation of Pareto surface.

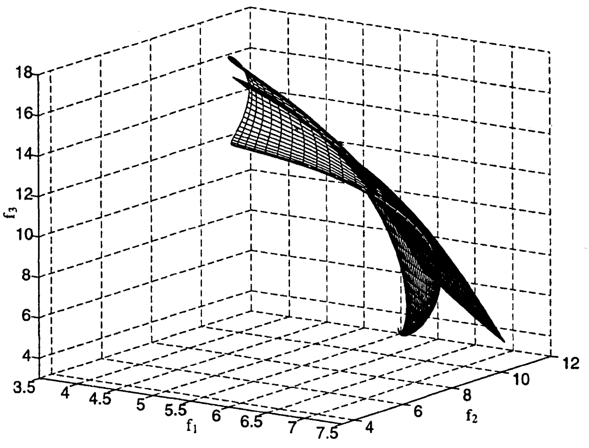

Fig. 7 Second-order representation of Pareto surface.

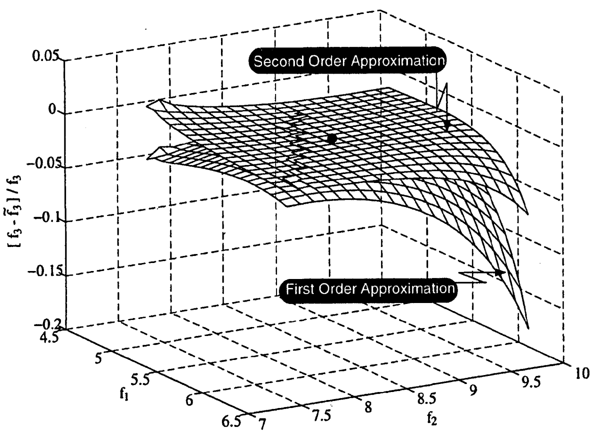

Fig. 8 Error in prediction.

range. To compare the accuracy of the Pareto surface approximations, the errors in f_3 predictions by first-order and second-order response surface approximations are compared at various values of f_1 and f_2. The results are shown in Fig. 8. From the error plot it can be seen that the performance of the second-order response surface is superior when compared to that of the first-order response surface.

Step 5

This is the most important step of the decision-making process. Note that in this research the Pareto designs are generated based on the aspiration levels. As a result, it is necessary to convert the DM's preferences into aspirations for obtaining a Pareto design that meets his or her preferences. If the DM is not satisfied with the current design and wants to explore the Pareto surface, the DM is expected to make both qualitative and quantitative decisions regarding the generation of a new Pareto point. Once the DM makes these decision, the aspirations that satisfy preferences are obtained such that they are on the Pareto surface approximation. To illustrate this step, consider the following three different scenarios of decision making. The goal here is to generate aspirations that meet the DM's requirements and lie on the approximate Pareto surface, i.e., attainable.

1) Improve f_1 by δ_1 (for example, 15%) while keeping f_2 constant and f_3 changes to a minimum.

2) Improve f_1 and f_3 as much as possible, and keep the changes in f_2 within δ_2 (for example, 15%).

3) Is it possible to improve f_1 at least by δ_1 (for example, 15%) and f_2 by δ_2 (for example, 15%) while keeping the changes in f_3 within δ_3 (for example, 20%)?

In scenario 1, the aspiration level for f_1 is given by $f_1^* - \delta_1$, and the aspiration for f_2 is given by f_2^*. The aspiration for f_3 can be obtained by substituting the aspirations of f_1 and f_2 into the second-order Pareto surface [Eq. (23)]. As a result, the aspirations that represent the DM's preferences in this scenario are obtained as $\bar{f} = [4.821\ \ 8.507\ \ 16.800]$. The new Pareto design generated by solving the CP problem, corresponding to these aspirations, is $f^* = [4.730\ \ 8.346\ \ 16.565]$. It can be seen from the result that the aspiration levels found using the second-order approximation reflect the DM's preferences, and the new Pareto design obtained is close to the aspirations, indicating that the Pareto surface approximation is accurate.

In scenario 2, there are an infinite number of possible aspirations for f_1 and f_3 because the amount of improvement specified by the DM is only qualitative. Here, it is assumed that any one of these aspirations is satisfactory. The aspiration levels are generated by solving the following optimization problem in the objective function space using the second-order Pareto surface [Eq. (23)]:

$$\begin{aligned} &\text{minimize} \quad 0.6 f_1 + 0.4 f_3 \\ &\text{subject to:} \quad f_3 - \tilde{f}_3 = 0.0 \\ &f_2^* + \delta_2 - f_2 \geq 0.0 \\ &\Delta f_i^L \leq \Delta f_i \leq \Delta f_i^U \end{aligned} \tag{24}$$

Note that the equality constraint on f_3 ensures that the aspirations predicted by solving the preceding optimization problem are on the second-order Pareto surface. It is necessary to solve the preceding optimization subject to move limits because it uses the approximation to the Pareto surface. The aspirations obtained by solving the preceding optimization problem within 30% move limits are given by $\bar{f} = [6.370\ \ 9.775\ \ 11.725]$. Note that these aspirations quantify the qualitative information given by the DM. The new Pareto solution, obtained by solving the CP problem, corresponding to these aspirations, is $f^* = [6.145\ \ 9.50\ \ 11.450]$, which is close to the aspirations specified and is one of the many Pareto designs that satisfies the DM's qualitative preferences.

In scenario 3, depending on the nature of the Pareto surface, there may or may not be a set of possible aspirations that will meet the DM's preferences. To answer the DM's question, the following optimization problem in objective function space is solved:

$$\begin{aligned} &\text{minimize} \quad f_1 \\ &\text{subject to:} \quad f_3 - \tilde{f}_3 = 0.0 \\ &f_1^* - \delta_1 - f_1 \geq 0.0 \\ &f_2^* - \delta_2 - f_2 \geq 0.0 \\ &f_3^* + \delta_3 - f_3 \geq 0.0 \\ &\Delta f_i^L \leq \Delta f_i \leq \Delta f_i^U \end{aligned} \tag{25}$$

This problem is solved within 30% move limits, and the results indicate that the problem has no feasible solution, i.e., the constraint on f_1 could not be satisfied. This means that it is not possible to improve f_1 by 15% while sacrificing f_3 by only 20%. However, the aspirations obtained by solving the problem without the constraint on f_1 are given by $\bar{f} = [5.320\ \ 7.231\ \ 17.015]$. The new Pareto solution, obtained by solving the CP problem, corresponding to these aspirations, is $f^* = [5.259\ \ 7.036\ \ 16.555]$, where f_1 is improved by 8%.

From these different scenarios, it is clear that the Pareto surface approximation approach can be used effectively in quantifying the decisions made by the DM. The aspirations predicted by the approximations, around the current Pareto point, are attainable, i.e., they are accurate and are approximately on the Pareto surface. This means that the new Pareto point obtained corresponding to these aspirations is close to the DM's preferences. This is important for the rapid convergence of the decision-making process and for the confidence of the DM in an interactive procedure.

Test Problem 2: High-Performance Low-Cost Structural Design

This is an application of IMOOP to a structural design and sizing problem. The problem is the design of a high-performance low-cost (HPLC) structure shown in Fig. 9. This problem was first introduced by Wujek.[27] It has been modified here to fit the multiobjective optimization framework. It includes the following objective functions:

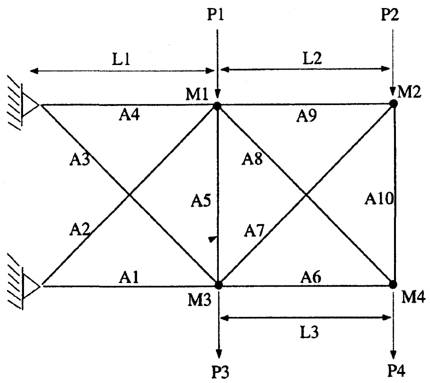

Fig. 9 HPLC structure.

minimize the structural weight W_{tot}, maximize the loads P_{1-4} the structure is capable of sustaining, and minimize the displacement of the structure, d. The goal of the optimization is to find the size and shape of a 10-bar truss that achieves a compromise among the three objective functions mentioned earlier while satisfying the design constraints on minimum payload and load requirements, as well as yield stress and first natural frequency constraints. The design vector in this problem is composed of the length of the rectangular first bay, L_1; and the top and bottom lengths of the outerbay, L_2 and L_3; the masses (payload) placed on all of the unconstrained nodes, M_{1-4}; and the areas of the truss members, A_{1-10}. The multiobjective optimization problem statement in standard form is

$$\text{minimize} \quad F(x) = \left\{ w_1 W_{tot}, \frac{w_2}{\sum P_i}, w_3 d \right\}$$

$$\text{subject to:} \quad g_1 = 1 - \frac{(M_{tot})_{min}}{\sum M_i} \geq 0$$

$$g_2 = 1 - \frac{(P_{tot})_{min}}{\sum P_i} \geq 0$$

$$g_3 = 1 - \frac{\omega_{1,min}}{\omega_1} \geq 0 \tag{26}$$

$$g_{4-13} = 1 - \frac{|(\sigma_{1-10})|}{\sigma_y} \geq 0$$

$$x^l \leq x \leq x^u$$

where $(M_{tot})_{min} = 5000$ lb, $(P_{tot})_{min} = 10{,}000$ lb, $\omega_{1,min} = 2.0$ Hz, $\sigma_{yield} = 14{,}000$ psi, and the constants w_i are chosen appropriately so that the objective functions are well scaled. The loads P_{1-4} applied to the structure are defined to be a function of the lengths of the bays L_{1-3} and the payload masses M_{1-4} placed on the structure. The displacement d is taken as the maximum absolute displacement of the top right node of the outerbay.

The following are the details of the implementation: The ideal point is set equal to zero for all of the objective functions. The initial aspirations (in the scaled space) are assumed to be $\bar{f} = [9.00 \;\; 12.00 \;\; 6.00]$. By solving the CP problem, a Pareto point that best represents the DM's initial preferences is obtained. The optimum objective function values are $f^* = [6.576 \;\; 8.768 \;\; 4.384]$. At the optimum all of the bay lengths L_{1-3}, the first and the second payloads M_1 and M_2, and two of the cross-sectional areas A_5 and A_{10} are at their lower bounds. The frequency constraint g_3 is the only design constraint that is active at the optimum.

The Pareto sensitivity analysis is conducted at the current point to study the effect of changes in one objective function on other objectives. Based on Eq. (11) the tradeoff matrix is obtained as

$$T = \begin{bmatrix} 1.00 & 0.000 & -0.880 \\ -1.455 & 1.000 & -0.652 \\ -0.546 & 0.000 & 1.000 \end{bmatrix} \tag{27}$$

From the tradeoff matrix it can be seen that a decrease in the weight or the displacement can be achieved without affecting the sustainable loads. This is because the bay lengths and the top payloads are at their lower bounds. The first-order approximation to the Pareto surface, which is determined based on the tradeoff matrix T [Eq. (27)], is given by

$$\tilde{f}_3 = 4.384 - [0.845 \;\; 0.465] \begin{bmatrix} f_1 - 6.576 \\ f_2 - 8.768 \end{bmatrix} \tag{28}$$

To fit a second-order response surface approximation, four approximate Pareto points in the neighborhood (within 20%) of the current Pareto design are used. The second-order Pareto approximation is

$$\tilde{f}_3 = 4.384 - [0.845 \;\; 0.465] \begin{bmatrix} f_1 - 6.576 \\ f_2 - 8.768 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} f_1 - 6.576 \\ f_2 - 8.768 \end{bmatrix}^T \begin{bmatrix} -0.172 & -0.018 \\ -0.018 & -0.334 \end{bmatrix} \begin{bmatrix} f_1 - 6.576 \\ f_2 - 8.768 \end{bmatrix} \tag{29}$$

Another way of obtaining this second-order Pareto information is by representing each objective and constraint in terms of the design variables using the second-order approximations. If one were to determine these second-order terms with respect to design variables, one would have to invoke 153 ($17 \times \frac{18}{2}$) function evaluations because the problem has 17 design variables. On the other hand, each projection mentioned earlier required an average of only 12 function calls. This means that the second-order Pareto surface is obtained by performing only 50 function evaluations. It is clear from this comparison that for problems that have large number of design variables and a few number of objectives the projection approach is computationally efficient.

In Fig. 10 the first-order approximation is compared with the second-order approximation. It can be seen from Fig. 10 that the displacement varies linearly with respect to the weight and varies nonlinearly with respect to the sustainable load. To compare the accuracy of these approximations, the error in f_3 predictions using first-order and second-order response surfaces are compared at different values of f_1 and f_2. The Pareto points at which the comparison is made are shown in Table 1 along with their perturbation (in %)

Table 1 Pareto points in the neighborhood of the current design

Function values			% Change from f^*		
f_1	f_2	f_3	$(f_1 - f_1^*)/f_1^*$	$(f_2 - f_2^*)/f_2^*$	$(f_3 - f_3^*)/f_3^*$
6.001	8.001	5.357	8.74	8.74	−22.19
5.019	6.693	8.033	23.67	23.67	−83.23
7.752	9.608	3.537	−17.87	−9.57	19.32
8.207	10.000	3.071	−24.79	−14.05	30.00
7.299	9.732	3.531	−11.00	−11.00	19.45
5.616	7.489	6.255	14.58	14.58	−42.67
5.530	9.975	4.904	15.90	−13.77	−11.86
7.635	8.852	3.669	−16.10	−0.96	16.31
7.545	10.000	3.330	−14.73	−14.05	24.04

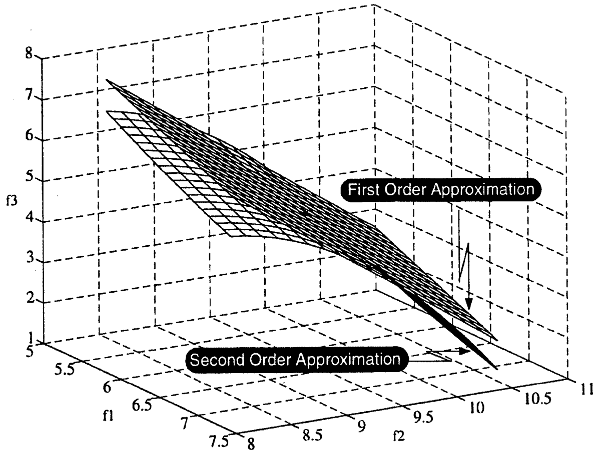

Fig. 10 Approximate representation of the Pareto surface.

Table 2 Performance comparison of the Pareto approximations

Exact	Linear approximation		Quadratic approximation	
f_3	$\tilde{f}_3$	% Error	$\tilde{f}_3$	% Error
5.357	5.227	2.41	5.360	−0.09
8.033	6.666	17.02	7.652	4.75
3.537	3.000	15.16	3.254	7.97
3.071	2.433	20.76	2.952	3.89
3.531	3.324	5.86	3.537	−0.17
6.255	5.790	7.43	6.165	1.45
4.904	4.706	4.02	5.022	−2.40
3.669	3.450	5.97	3.549	3.27
3.330	2.993	10.10	3.348	−0.58

from the current Pareto design. The predictions, i.e., $\tilde{f}_3$, and the error in predictions at these Pareto points are shown in Table 2. From these results it can be concluded that the performance of the second-order approximation is superior compared to that of the first-order approximation. As mentioned earlier, the DM can conduct tradeoff and decision-making analysis to change preferences by using the second-order Pareto surface approximation.

Conclusions

An IMOOP that is based on a weighted Tchebyshev norm, i.e., aspiration-level approach, is developed. This procedure facilitates the designer's involvement as a DM in the design of multiobjective systems. This method provides the DM with a formal means for efficient design exploration around a given Pareto point. More specifically, the method provides the DM with the first-order Pareto sensitivity information and the Pareto surface approximation at a given Pareto design for decision making and tradeoff analysis.

A method based on the active constraint set is developed for calculating the first-order Pareto sensitivity information. A response surface methodology is used for approximating the Pareto surface. A second-order response surface approximation is used. The first-order terms of the approximation are determined based on the Pareto sensitivity information, and the second-order terms of the approximation are determined based on the approximate Pareto data generated in the neighborhood of the current design. The approximate Pareto data are obtained by projecting the aspirations that are on the first-order Pareto surface approximation onto the actual Pareto surface.

The IMOOP has been successfully applied to two test problems. The first problem consists of a set of simple analytical expressions for its objective and constraints. The second problem is the design and sizing of an HPLC 10-bar structure that has multiple objectives. The results indicate that the Pareto designs obtained by using the Pareto surface approximation are reasonable and that the performance of the second-order approximation is superior compared to that of the first-order approximation. Using this procedure, the aspirations that reflect the DM's preferences can easily be generated, and the actual Pareto point obtained by using these aspirations is close to the aspirations themselves. This is important for the confidence of the DM when using the IMOOP for obtaining a satisfactory final Pareto design in a minimal number of iterations.

Acknowledgment

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